

Local Realistic Theory for PDC Experiments Based on the Wigner Formalism

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In this article we present a local hidden variables model for all experiments involving photon pairs produced in parametric down conversion, based on the Wigner representation of the radiation field. A modification of the standard quantum theory of detection is made in order to give a local realistic explanation of the counting rates in photodetectors. This model involves the existence of a real zeropoint field, such that the vacuum level of radiation lies below the threshold of the detectors.

Key words: Parametric Down Conversion; Wigner Representation; Zeropoint Field; Local Realism; Bell's Inequalities.

1. Introduction

Following Bell, a local hidden variables model (LHV) exists for an EPR experiment if it is possible to write the single and joint detection probabilities in the form [1]

$$\begin{aligned} P_i &= \int \rho(\lambda) P_i(\lambda, \phi_i) d\lambda; \\ P_{12} &= \int \rho(\lambda) P_1(\lambda, \phi_1) P_2(\lambda, \phi_2) d\lambda, \end{aligned} \quad (1)$$

λ being the hidden variables with a probability distribution $\rho(\lambda)$. $P_i(\lambda, \phi_i)$, $i = 1, 2$, are some functions fulfilling the conditions $0 \leq P_i(\lambda, \phi_i) \leq 1$. ϕ_1 and ϕ_2 represent controllable parameters of the experimental setup.

Many experiments have been performed in order to test local realism vs. quantum mechanics via Bell's inequalities, but none of these experiments have violated Bell's genuine inequalities, which are based upon the assumptions of realism and locality alone, but rather inequalities which involve additional assumptions, like *no-enhancement*. In particular, experiments using photon pairs produced in parametric

down conversion (PDC) have become very popular in the last few years [2]. LHV models for EPR experiments with PDC have appeared in the literature [3], the aim of which has been to show that local realism has not been ruled out by these experiments, and to stress the relevance of the so called "loopholes", in particular the one due to the low efficiency of optical photon counters. However, those models were mathematical constructs without physical content; they have no predictive power.

The purpose of this paper is to exhibit a physical model which is based on the Wigner formulation of PDC. Our previous work [4] has shown that the Wigner function of the electromagnetic field is positive for all performed experiments, and hence it provides a realistic description for the *production* and *propagation* of this kind of radiation in terms of the coupling between the zeropoint field and the laser beam. The expression for the PDC field in the Wigner representation is

$$\begin{aligned} E^{(+)} &= \sum_k \left(\frac{\hbar\omega}{\epsilon_0 L_0^3} \right)^{1/2} [\alpha_k e^{-ik \cdot r + i\omega t} \\ &+ g \alpha_k^* e^{-i(k_0 - k) \cdot r + i(\omega_0 - \omega)t} + \frac{1}{2} g^2 \alpha_k e^{-ik \cdot r + i\omega t}], \end{aligned}$$

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where the first term is the zeropoint field (ZPF) that crosses the crystal without any modification, and the other terms are produced via the non-linear coupling between the laser and the ZPF (g is the coupling parameter). The Wigner function corresponding to the zeropoint radiation is the gaussian

$$W(\{\alpha_k^*, \alpha_k\}) = \prod_k \frac{2}{\pi} e^{-2|\alpha_k|^2}. \quad (2)$$

On the other hand, the single and joint detection probabilities in PDC experiments are given by

$$P_s = \int W(\{\alpha_k\}, \{\alpha_k^*\}) Q(\{\alpha_k\}, \{\alpha_k^*\}, \phi) d^N \alpha_k d^N \alpha_k^*, \quad (3)$$

$$P_c = \int W(\{\alpha_k\}, \{\alpha_k^*\}) Q_1(\{\alpha_k\}, \{\alpha_k^*\}, \phi_1) \times Q_2(\{\alpha_k\}, \{\alpha_k^*\}, \phi_2) d^N \alpha_k d^N \alpha_k^*, \quad (4)$$

where

$$Q = \frac{\eta}{h\nu} [\tilde{I}(\{\alpha_k\}, \{\alpha_k^*\}, \phi) - \tilde{I}_0] \quad (5)$$

$$= \frac{\eta}{h\nu} \int dt \int d^2 r [I(\{\alpha_k\}, \{\alpha_k^*\}, \phi, \mathbf{r}, t) - I_0].$$

$I = E^+ E^-$ is the intensity of the field and I_0 is the mean intensity of the the ZPF. The integration is carried over the time window and the surface aperture of the detector. We have divided (5) by the typical energy of one “photon”, so that Q becomes dimensionless, η being the quantum efficiency of the detector.

The relevant question is whether (4) may be considered a particular case of (1). Actually the answer is not affirmative because we cannot guarantee the positivity of Q . Consequently we conclude that it is not possible to interpret directly the Wigner-function formalism as an LHV model for the PDC experiments.

2. The Detection Model

We shall devote the rest of the article to the description of the model of a detector that works in a strictly local way. We will first show the basic points of our model:

1.) The detector is formed by a set of individual photodetector elements, D_j , each characterized by a frequency ω_j , and a wave vector \mathbf{k}_j ($\omega_j = |\mathbf{k}_j|/c$), to

which D_j responds. We shall consider the direction of \mathbf{k}_j to be normal to the surface of the detector, which is taken as a cylinder of area πR^2 and length L .

The photodetector element D_j is sensitive to radiation with frequencies in the interval $(\omega_j - \Delta\omega/2, \omega_j + \Delta\omega/2)$ with $\Delta\omega \approx 2\pi/T$, T being the detection time window. If we assume that the incoming light beam has frequencies in the interval $(\omega_{\min}, \omega_{\max})$ with an average frequency $\bar{\omega} = (\omega_{\max} + \omega_{\min})/2$, and τ is the coherence time of the beam, we shall have $\delta\omega \equiv \omega_{\max} - \omega_{\min} \approx 2\pi/\tau$, so that the minimum number of detecting elements is $N \approx \delta\omega/\Delta\omega \approx T/\tau$. By putting typical values, $T = 10^{-8}$ s and $\tau = 10^{-12}$ s, we have the condition $N > 10^4$.

2.) The relevant quantity for the detection is a filtered field corresponding to a detector element D_j :

$$\bar{E}_j^{(+)} = \frac{1}{\pi R^2 L T} \int_V dV \int_0^T E^{(+)}(\mathbf{r}, t) e^{-i\mathbf{k}_j \cdot \mathbf{r} + i\omega_j t} dt. \quad (6)$$

3.) We now define the *effective intensity* obtained from the filtered fields in the form

$$\bar{I} = c\epsilon_0 \sum_{j=1}^N \bar{E}_j^{(+)} \bar{E}_j^{(-)}. \quad (7)$$

After that we replace (5) by the expression

$$Q(\bar{I}) = (1 - e^{-\zeta(\bar{I} - \bar{I}_0)}) \Theta(\bar{I} - I_m); \quad \zeta = \eta(h\nu)^{-1}, \quad (8)$$

which completes the definition of our model. \bar{I}_0 is the average of \bar{I} for the ZPF. I_m is some threshold intensity fulfilling the condition $I_m > \bar{I}_0$, and $\Theta(x)$ is the Heaviside function.

4.) Now we rewrite the detection probabilities in the following equivalent form

$$P_s = \int \rho(\bar{I}) Q(\bar{I}) d\bar{I}; \quad (9)$$

$$P_c = \int \rho_{12}(\bar{I}_1, \bar{I}_2) Q_1(\bar{I}_1) Q_2(\bar{I}_2) d\bar{I}_1 d\bar{I}_2.$$

The probability distribution for the *effective intensity* is a gaussian which is determined by the mean and the standard deviation (for details see [5]). For instance, the probability distribution for the zeropoint field (making $\beta_k \rightarrow \alpha_k$) is

$$\rho(\bar{I}_0) = \frac{1}{\sqrt{2\pi}\sigma_0} e^{(\bar{I}_0 - \langle \bar{I}_0 \rangle)^2 / 2\sigma_0^2},$$

$$\langle \bar{I}_0 \rangle = \frac{\bar{\omega}\delta\omega}{8\pi cL}; \quad \sigma_0 = \langle \bar{I}_0 \rangle \sqrt{\frac{\tau}{T}}. \quad (10)$$

The probability distribution of the effective intensity when there is a PDC signal present, $\rho(\bar{I})$, is similar to that of the ZPF, but the following remarks are in order: (a) We shall define the signal mean effective intensity by $\bar{I}_s = \bar{I} - \langle \bar{I}_0 \rangle$. If we assume that the signal enters parallel to the axis of the detector, it can be shown that the “filtered intensity”, $\langle \bar{I}_s \rangle$, is equal to the actual intensity, i. e. $\langle \bar{I}_s \rangle \approx \langle I_s \rangle$. In practical situations the relation $\langle \bar{I}_s \rangle \ll \langle \bar{I}_0 \rangle$ is fulfilled. (b) The relation between the mean and the standard deviation is completely analogous to that of the ZPF alone (see (10)), and the distribution may be written

$$\rho(\bar{I}) = \frac{\sqrt{T/\tau}}{\sqrt{2\pi}\langle \bar{I}_0 \rangle} e^{-(\bar{I} - \langle \bar{I}_0 \rangle - \langle \bar{I}_s \rangle)^2 T / 2\langle \bar{I}_0 \rangle^2 \tau}. \quad (11)$$

On the other hand, $\rho(\bar{I}_1, \bar{I}_2)$ is a double gaussian function which is defined by the mean values of its marginals, their standard deviations and the correlation function $\langle (\bar{I}_1 - \bar{I}_{1s} - \bar{I}_{10})(\bar{I}_2 - \bar{I}_{2s} - \bar{I}_{20}) \rangle$.

3. The Detection Probabilities

In this section we shall compare the predicted detection probabilities of our model with those of quantum optics. In the case of single detection probability, let us consider the three following possible situations:

i) $\langle \bar{I}_s \rangle = 0$. In sharp contrast with quantum optics, our model predicts the existence of some counts in any detector even in the absence of signal, the probability being very small if $I_m - \langle \bar{I}_0 \rangle \gg \sigma_0$.

ii) $\zeta \langle \bar{I}_s \rangle \ll \sigma_0$. This should be the normal situation in experimental practice. Also, we may choose I_m so that $\langle \bar{I}_0 \rangle + \langle \bar{I}_s \rangle - I_m \gg \sigma_0$, but $I_m > \langle \bar{I}_0 \rangle$ in order to preserve the positivity of Q in (8). With these two approximations, it can be shown that $P_s^m \approx \zeta \langle I_s \rangle$, a result that coincides with the quantum result.

iii) $\zeta \langle \bar{I}_s \rangle \gg \sigma_0$. In this case the detector saturates and gives a count in every time window.

Finally, in the case of the joint detection probability, the predictions of our model coincide with the quantum result in the limits $\eta_i(\bar{I}_i - \langle \bar{I}_{i0} \rangle)/h\nu_i \ll 1$ and $\langle \bar{I}_{is} \rangle + \langle \bar{I}_{i0} \rangle - \bar{I}_{im} > \sigma_0$.

4. Constraints of the Model

In our model there is a trade-off between the constraint $\langle \bar{I}_0 \rangle + \langle \bar{I}_s \rangle - I_m \gg \sigma_0$, required for the linearity of the response at low efficiency, and $I_m - \langle \bar{I}_0 \rangle \gg \sigma_0$, needed for the smallness of the dark counting probability. The need to satisfy both conditions implies that $\langle \bar{I}_s \rangle \gg \sigma_0$. This means that there is a minimal intensity of the signal which may be reliably detected, a constraint absent in the quantum theory of detection, but certainly existing in experimental practice.

Let us analyze the consequences of the constraint. In experimental practice a lens is placed in front of the detector in such a way that the signal field has spatial coherence on the surface of the lens. The condition for having spatial coherence is $d\lambda \geq R_l R_C$, d being the typical distance between the nonlinear medium (with radius R_C) and the detector; R_l is the radius of the lens. On the other hand, the zero-point field is not modified by the lens, which is evident because of the fact that energy cannot be extracted from the vacuum. As a consequence the intensity of the incident signal is amplified by a factor $b^2 \equiv \pi^2 R_l^4 / \lambda^2 f^2$, f being the focal distance. On the other hand, 84 % (91 %) of the total intensity is concentrated within the first (second) ring of the diffraction pattern, with a radius $R = a \times (f\lambda/2R_l) = a\lambda/A_r$, where $A_r = 2R_l/f$ is the relative aperture of the lens and $a = 1.22$ (2.23) for the first (second ring). Consequently, the optimum radius of the detector is given by R .

By taking into account the above considerations, and using (10) and the condition $d\lambda \geq R_l R_C$, the constraint $\langle \bar{I}_s \rangle \gg \sigma_0$ gives

$$\text{Rate} \gg \frac{\eta f^2 R_C^2}{2Ld^2\lambda\sqrt{\tau T}}, \quad (12)$$

an expression that puts a lower bound on the single rates which may be used in reliable experiments. This result cannot be derived from (conventional) quantum theory. By putting typical parameters, that is $\eta \approx 0.1$, R_l , L and f of the order of fractions of a centimeter, $T \approx 10$ ns, $\lambda \approx 700$ nm and $\Delta\lambda \approx 10$ nm (which gives a coherence time $\tau \approx 1$ ps) we get a minimal counting rate of the order of $10^5 - 10^6$ counts per second. This figure agrees fairly well with the actual experiments and should be considered a success of our model.

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